

Can Signature Biometrics Address both Identification and Verification Problems?

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Abstract—Handwritten signatures are one of the most socially acceptable and traditionally used person identification and authentication metric. Although a number of authentication systems based on handwritten signatures have been proposed, a little attention is paid towards employing signatures for person identification. In this work, we address both the identification and verification problems related to analysis of dynamic handwritten signatures. In this way, the need to present username before biometric verification can be eliminated in current signature based biometric authentication systems. A compressed sensing approach is used for user identification and to reject a query signature that does not belong to any user in the database. Once a person is identified, an automatic alignment of query signature with the reference template is carried out such that the correlations between two signature instances are maximized. An elastic distance matching algorithm is then run over the presented data which declares the query signature as either genuine or forged based on the dissimilarity with the reference signature. Our results show that dynamic signatures can be accurately used for person identification along with the traditional verification methods.

Keywords—Sparse representation, Handwritten signatures, Compressed sensing, Canonical correlation, Distance matching.

I. INTRODUCTION

Biometrics have found wide applications in person identification and verification (I&V). Biometrics are characteristic of its owner and are hard to forge or duplicate. Currently, biometric systems based on physiological biometric traits such as palm print, face, finger print, ear, body odor, hand geometry, iris etc are predominant. Physiological biometrics are known to deal with both the I&V problems adequately with high success rates. On the other hand, behavioral traits such as handwritten signatures, voice, lip motion and gait have relatively low accuracy rates [1]. The need is to investigate how behavioral biometrics based schemes can be brought at par with their counterparts belonging to physiological biometric traits.

Among behavioral biometrics, handwritten signatures are a lucrative tool for person authentication because of their social acceptability, ease in data collection and less computational resource requirements. Smart phones and modern hand-held devices with interactive interfaces pose a huge market for signature based authentication systems. Such systems are free from illumination artifacts, pose variations and high computational load unavoidable in their prominent counterpart from physiological biometrics – face recognition based systems. However, the flip side of existing signature based systems is

that they are solely focused on identity verification problem [2] [3]. In contrast, face recognition based systems can tackle both I&V problems simultaneously with high accuracy [4]. This paper is an effort to address this shortcoming of signature biometrics. We want to design a system that can suffice both identification and authentication needs with high accuracy. In addition, the proposed scheme must be computationally feasible and scalable to handle large databases.

Signatures I&V can be run in two modes; either static (off-line) or dynamic (on-line). Offline signatures consists of an image and static properties of signatures are analyzed during the authentication process. In contrast, dynamic signatures consist of time series of data values that give important information about the signature speed, jerks, pressure, acceleration, angle of curvatures, elevation and varying pressure values that are associated with the writer's hand movements along the signature contour. Both local and global information is analyzed in this way and a more secure, robust and efficient verification system can be realized [5]. We have used online signatures in the current work due to their easy deployment on mobile devices and higher verification rates.

Identification of dynamic handwritten signatures is a challenging problem. This is due to the reason that even two genuine signature instances from same user have variations in their dynamical properties. This behavior is unique from other famous biometric traits (like face, finger prints, iris etc.) whose variability is limited and often occurs over long periods of time. Moreover, signatures belonging to two different users may look similar in texture. In order to tackle these problems we have extracted a number of useful features that help in discriminating two similar signatures. As an example, two signatures form different people may look similar in shape but it is highly unlikely that the dynamical information associated with signature pattern will also be alike. We have used compressed sensing based sparse representation to tackle the challenging signature identification problem.

Compressed sensing provides a robust and computationally less intensive solution to signature recognition problem. It has also been used successfully for face recognition and has out performed state-of-the-art [6]. From the deployment point of view, many practical advantages have also accompanied the sparse representation based identification (SRI). Such a representation is robust towards noise and intrinsic variation in signature instances. These recognition schemes are also scalable both in terms of features and users [7].

We have empirically evaluated logical questions associated with SRI. The amount of data required for training is a major concern in any practical system. Therefore, we analyze how SRI performance shape up with the number of training samples. Another important metric for gauging system performance is its robustness towards random forgeries, which is reported in section III-E. Next, we combine recognition algorithm with a separate verification mechanism to detect skilled forgeries. This verification mechanism combines canonical correlation analysis (CCA) and dynamic time warping (DTW) to achieve a robust authentication framework.

Upto the best of our knowledge, we are the first to apply compressed sensing principles for signature based user identification. The verification problem is addressed using a two level authentication mechanism based on CCA and DTW. The technique is tested on two large publicly available signature datasets. The resulting system is highly accurate in user identification on both the datasets. Verification performance achieved is also among the best results reported on these datasets. In this way, we propose a scheme that uses dynamic signatures to first identify the correct user and then verify its authenticity without requiring any other information such as user name, keyword or identification number.

II. METHODS

A. Problem Formulation

The problem is to correctly identify and verify a signature presented by the user. We will like to formally define both the problems:

Definition 1 (Signature Identification). *Given a dynamically gathered handwritten signature, we want to correctly identify the user from a maintained database, to which the query signature belongs.*

Definition 2 (Signature Verification). *Having established that the query signature belongs to some user, we want to verify whether it is a genuine signature or an attempt of forgery.*

We propose a hierarchical approach to address both the above mentioned problems (see figure 1).

B. Signature Recognition

1) *Feature Space*: Recognition is performed in a feature space. The main requirement of SRI is that the length of all signature instances must be equal in feature space. This requirement can be easily fulfilled in the case of face recognition by taking images of only fixed resolution. However in case of handwritten signatures, different instances vary in length and they must be converted to a fixed length before performing SRI. This fixed length remapping must be in a lower dimensional space so that the recognition task may not become computationally intensive. Here re-sampling works poorly since it introduces small errors in signature contour that can mislead the system in case of skilled forgery attacks.

In compressed sensing, the role of specific features is not important, rather their number is of more relevance. We have selected random subspace mapping to generate feature space because sparse recognition performs well even for random features [6]. Such a mapping helps in achieving equal length

feature vectors with significantly reduced dimensionality. This makes the further processing less computationally intensive without undermining the recognition accuracy. Random projections are used for random feature extraction. To ensure that the random basis are orthogonal to each other, Gram-Schmidt orthogonalization procedure is used for QR decomposition of random matrix such that: $\mathbf{R} = \mathbf{R}_Q \mathbf{R}_R$. $\mathbf{R}_{m \times n}^{-1}$ is a random matrix whose i.i.d elements are chosen from a normal distribution with mean 0 and variance $1/m$ and \mathbf{R}_Q is the orthogonal matrix. This orthogonality ensures that correlations between axes are minimized such that:

$$\langle r_i, r_j \rangle = \delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases} \quad (1)$$

2) *Compressed Sensing*: Compressed sensing is a newly emerged theory that was primarily used in signal compression, reconstruction and coding. It has also been applied to pattern recognition, reconstruction, medical imaging devices and wireless sensing networks. The results are strikingly superior or atleast competitive with state-of-the-art in most cases. Compressed sensing theory describes how a signal can be reconstructed from very few samples, surpassing the limits set by Nyquist-Shannon theorem. It exploits the property that a sample of sparse signal can be expressed in the form of a linear function that is applied on that sparse signal [8]. We propose that such a sparse representation can tackle the recognition problem of handwritten signatures with noise, corruption and sample-to-sample variation.

Compressed sensing theory demands that the signal to be recovered must be sparse. A signal $\mathbf{x} \in \mathbb{R}^n$ is said to be t -sparse if it can be represented by few ($t \ll n$) nonzero elements when mapped on an orthonormal basis. The other coefficients may be strictly or approximately zero. Compressed sensing also requires incoherence between the sensing basis and representation basis so that least number of elements may represent the sensed signal. The random subspace mapping using orthogonal basis as discussed in section II-B1 is well suited for the purpose and can be used to obtain good recognition rates.

Suppose the t -sparse signal \mathbf{x} has support: $\mathcal{I} = \{i : x(i) \neq 0\}$ with cardinality $|\mathcal{I}| = t$. We are given a sampled version \mathbf{y} of \mathbf{x} , and the function that samples \mathbf{x} in the form of m linear combinations is represented by $\mathbf{A}^{m \times n}$.

$$\mathbf{y} = \mathbf{A}\mathbf{x}, \quad \because y_j = \langle \mathbf{a}_j, \mathbf{x} \rangle \mid j = 1, 2, \dots, m \quad (2)$$

For the purpose of sampling, the sensing matrix \mathbf{A} needs to have its columns nearly orthogonal. For this purpose we have done random subspace mapping of the signature data so that the uniform uncertainty principle (UUP) can be met.

The sensing function \mathbf{A} is a dictionary of training signature instances that takes the form of a matrix with columns belonging to training signatures from users. Each group of consecutive training signatures belong to an individual user; such that for k users, \mathbf{A} takes the form $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_k]$.

¹Sans-Serif capital letter R represents random matrix. Bold capital letter (like \mathbf{X}) indicates a matrix while bold small letter (like \mathbf{x}) is used to represent vector. When respective values of a vector or matrix are indicated, normal italic alphabets (like x_i) are used. $|\cdot|$ represents cardinality of finite set. $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_F$ denote ℓ_1 , ℓ_2 and Frobenius norms respectively. $\mathbf{A} \circ \mathbf{B}$ is used to denote Hadamard matrix product. δ symbolizes the kronecker delta.

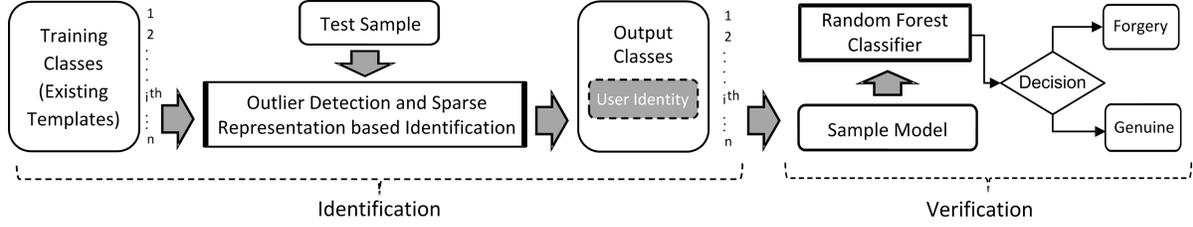


Fig. 1. An overview of proposed scheme to address Identification and Verification problems for Dynamic Handwritten Signatures.

The probe sample is also represented as a column vector. This vector is created by stacking the feature vectors one above the other. Random projections are used to covert high dimensional probe and training signatures to an equi-length, lower dimensional form. The system of equations is then solved for \mathbf{x} and the best match is declared class of probe signature.

The Eq.2 represents an ill posed system of linear equations when $m < n$. Therefore, the number of unknowns would be less than the available equations. The task at hand is to recover sparse signal \mathbf{x} when sampling function \mathbf{A} and sampled version \mathbf{y} are known. For our problem, recovered signal \mathbf{x} is either strictly sparse (when the testing instance also lie in the training set) or nearly sparse (when the training instance does not lie in the training set). The solution to sparse signal recovery problem is possible by ℓ_1 norm minimization when \mathbf{x} is sparse [9]. For noisy sparse signal recovery, this problem can be represented as:

$$\min \|\hat{\mathbf{x}}\|_1; \text{ such that } \|\mathbf{A}\hat{\mathbf{x}} - \mathbf{y}\|_2 \leq \epsilon \quad (3)$$

where ϵ is the permissible deviation in the signature data and \mathbf{y} is a test image that belongs to some user class, say i^{th} . This convex optimization problem can be solved by linear optimization techniques.

We have used l1_ls matlab solver [10] to solve the noisy sparse signal recovery problem defined in Eq. 3. This package solves ℓ_1 regularized least squares problems using truncated Newton's interior point method [11]. The regularization parameter was set to 0.5 in all the experiments. This package is customized to solve large scale problems and has relatively lower storage requirements. Since the signature instances are mapped on orthogonal random basis, the mutual coherence of dictionary \mathbf{A} is small. This ensures that the off-diagonal elements of $\mathbf{A}^T \mathbf{A}$ are nearly zero and the optimization routines converge fast. After running tests, we have found that the average number of preconditioned conjugate gradient (PCG) steps required to solve recognition problem on SVC and SigComp datasets are 53 and 60 respectively.

C. Signature Verification

1) *Canonical Correlation Analysis*: CCA finds the appropriate coordinate system in which the correlations are maximum. It attempts to transform the basis of two input vectors such that the mutual linear correlations between zero-centered variables are maximized in the transformed domain. The case of signature verification is suitable for application of CCA. This is due to the reason that mutual information between two samples (\mathbf{x} and $\hat{\mathbf{x}}$) of same signature (whether

the attempt is of random or skilled forgery) is not zero i.e. $\mathcal{I}(x, \hat{x}) \neq 0$ [12]. If we let two signature instances with n features belonging to same user be $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n] \in \mathbb{R}^u$ and $\hat{\mathbf{X}} = [\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \dots, \hat{\mathbf{x}}_n] \in \mathbb{R}^v$, CCA tries to find $\mathbf{W}_x \in \mathbb{R}^u$ and $\mathbf{W}_{\hat{x}} \in \mathbb{R}^v$ such that the correlation between two signature instances get maximized and distance is minimized when \mathbf{X} and $\hat{\mathbf{X}}$ are mapped onto \mathbf{W}_x and $\mathbf{W}_{\hat{x}}$ respectively,

$$\mathbf{X}' = \mathbf{W}_x^T \mathbf{X} \quad \text{and} \quad \hat{\mathbf{X}}' = \mathbf{W}_{\hat{x}}^T \hat{\mathbf{X}} \quad | \quad \min \|\mathbf{X}' - \hat{\mathbf{X}}'\|_F \quad (4)$$

We have used a generalized eigenvalue representation of CCA to solve problem presented in equation 4. This representation is given by:

$$\begin{bmatrix} 0 & C_{x\hat{x}} \\ C_{\hat{x}x} & 0 \end{bmatrix} \mathbf{W} = \lambda \begin{bmatrix} C_{xx} & 0 \\ 0 & C_{\hat{x}\hat{x}} \end{bmatrix} \mathbf{W} \quad (5)$$

In this way CCA is used to neutralize affine transformations between different instances of signatures. This means that strong linear relationships between two signature instances will be taken into account despite variations in coordinate system. It removes the need of preprocessing steps in the verification pipeline and helps in achieving high performance.

D. Distance Matching

After aligning the two signatures using CCA, an elastic time warping technique is used to calculate the distance so that a measure of similarity can be obtained. We have used Dynamic Time Warping (DTW) as a non-metric distance measure for verifying signatures. It dynamically warps the time axis to achieve a best alignment between two signature sequences. The distance scores with the ensemble of training signatures are calculated, normalized with statistical parameters and passed on for classification (see section III-C). Interested readers can find details of DTW in [13].

III. EVALUATION

A. Datasets:

We have tested our scheme on two large databases collected as part of verification campaigns namely SVC 2004 [14] and SigComp 2011 [15].

1) *SVC 2004*: It consists of Chinese and English signatures data for two tasks performed by 100 users. The first task includes only x, y coordinates and pen-up points. More detailed information including elevation and pressure signals are included in second task. Each user data consists of 40 signatures among which 20 are genuine and 20 are skilled forgeries. All signatures are collected at 100 Hz using WACOM Intuos tablet.

TABLE I. LOCAL FEATURES

Feature Name	Formula
Sample Number n	$n = \{1, 2, 3, \dots, N\}$
Time Stamp t	$t = \{t_{\text{start}} \dots t_{\text{end}}\} = \{t_1 \dots t_N\}$
X and Y Cartesian Coordinates	$\{x_n, y_n\}_{N \times 2}$
Speed in X Direction	$\{s_n^x\} = \{\dot{x}_n\}/\{t_n\}$
Speed in Y Direction	$\{s_n^y\} = \{\dot{y}_n\}/\{t_n\}$
Root Mean Square Speed:	$ s_n = \sqrt{s_n^x{}^2 + s_n^y{}^2}, \quad \forall n \in [1, N]$
Acceleration in X Direction	$\{a_n^x\} = \{\ddot{x}_n\}/\{t_n\}$
Acceleration in Y Direction	$\{a_n^y\} = \{\ddot{y}_n\}/\{t_n\}$
Tangential Acceleration a_{t_n}	$ a_{t_n} = \dot{s}_n = \text{dif} \left(\sqrt{s_n^x{}^2 + s_n^y{}^2} \right)$
Centripetal Acceleration a_{c_n}	$ a_{c_n} = s_n \cdot \dot{\theta}_n, \quad \forall n \in [1, N]$
root mean square Acceleration	$ a_n = \sqrt{a_{t_n}{}^2 + a_{c_n}{}^2} \quad \forall n \in [1, N]$
Path Tangent Angle	$\theta_n = \tan^{-1}(\frac{y_n^x}{x_n^x}), \quad \forall n \in [1, N]$
Log of Radius of Curvature	$\delta_n = \log(\frac{r_n}{\theta_n}), \quad \forall n \in [1, N]$
Perpendicular Component of Curve	$p_c = \sin(\theta_n)$
Horizontal Component of Curve	$h_c = \cos(\theta_n)$
Pressure	$p = \{p_1, p_2, p_3 \dots p_N\}$

2) *SigComp 2011*: Both Chinese and Dutch signatures data is included that consists of x, y and z coordinates. Chinese dataset contains 960 signatures while 1790 signatures are present in dutch dataset. Chinese subcorpus includes data from 20 users and dutch subcorpus includes data from 64 users. All signatures are collected at 200 Hz using WACOM Intuos3 A3 Wide USB Pen Tablet.

B. Dynamic Feature Extraction

Online signature phenomenon has made signature verification systems at par with the other biometrics. In contrast with the traditional off-line based signature matching techniques, online signature comparison is a more secure and reliable biometric. The changing dynamic properties of a signature are extremely difficult to imitate correctly. We have extracted a number of dynamic features that are listed up in table I. Derivatives are of great significance in capturing the dynamical features of handwritten signatures. We have also included first and second order derivatives in our feature set.

$$\mathcal{F} = \{t, x_n, y_n, s_n^x, s_n^y, |s_n|, a_n^x, a_n^y, |a_{t_n}|, |a_{c_n}|, |a_n|, \theta_n, \delta_n, p_c, h_c, p\}$$

$$\hat{\mathcal{F}} = \{\mathcal{F}, \dot{\mathcal{F}}, \ddot{\mathcal{F}}\}$$

The derivatives are calculated using second order regression instead of simple time series difference so that a more useful changing pattern can be obtained.

C. Enrollment & Dictionary Compilation

During the enrollment phase, authentic signatures are gathered from subjects for training. These signatures capture the general variety that is expected in future instances supplied for validation. For the purpose of recognition, all features are normalized and then compiled in the form of a vector to create a dictionary as described in section II-B2. For the verification purpose, we have used an approach—similar as [3]—to capture the differences in signatures collected during enrollment phase. The distinguishing feature of our scheme is that instead of a three dimensional normalized feature vector, we have used statistical measures like mean and variance to account for the variation in training feature set. Our approach is more

robust towards variation among acquired reference signatures and gives a small feature set of distance measures that can be classified at a fast rate. Let we have got ' N ' signatures from a data-set as enrollment data:

$$\mathcal{E} = \{\forall s_n \mid n \in [1, N]\}$$

The first step is to calculate distance between all unique pairs of enrollment signatures:

$$\mathcal{M}_i = \{\forall \mathcal{D}(s_i, s_j) \mid i \neq j \wedge j \in [1, N]\} \quad \forall i \in [1, N]$$

where the distance function is commutative, i.e. $\mathcal{D}(s_i, s_j) \equiv \mathcal{D}(s_j, s_i)$. Next, signature with minimum average distance with all other $(N - 1)$ enrolled signatures is selected as reference:

$$\mathcal{M}_{ref} = \underset{i}{\operatorname{argmin}} \left(\frac{1}{N-1} \sum_{k=1}^{N-1} \mathcal{M}_i(k) \right)$$

The distance between query signature and reference is calculated after applying CCA to maximize mutual correlations between the two patterns:

$$\mathcal{M}_{query} = \{\forall \mathcal{D}(\mathcal{F}_{cca}(s_{query}, s_i)) \mid i \in [1, N]\}$$

Two simple statistical features are extracted from each set of distances calculated for a test signature s_{query} . These are mean and variance:

$$\mu_i = \frac{1}{N-1} \sum_{k=1}^{N-1} \mathcal{M}_i(k); \quad \sigma_i^2 = \frac{1}{N-1} \sum_{k=1}^{N-1} (\mathcal{M}_i(k) - \mu_i)^2 \quad \forall i$$

The distance measure feature set (\mathcal{V}) comes out to be:

$$\mathcal{V} = [\mathcal{M}_{query} \circ 1/\{\mu\}_{1 \times N}, \mathcal{M}_{query} \circ 1/\{\sigma^2\}_{1 \times N}]$$

D. Recognition & Verification

For recognition, SRI is carried out for all test instances using dictionary \mathbf{A} and ℓ_1 norm minimization. For verification, a Random Forest classifier is used for classifying genuine and forged signatures. This classification algorithm creates an ensemble of trees and then decides the input class using the votes from each tree. A standard 10 fold cross validation is used so that the verification results may not get biased due to nature of input.

E. Results & Observations

The I&V performance is evaluated on skilled forgeries which are difficult to detect as compared to random forgeries. Ten signatures are used for training on both the datasets. The results outlining the performance of our scheme are listed in table II. For the sake of comparison, table III enlists state-of-the-art performances on SVC and SigComp datasets. I&V accuracies (\mathcal{A}_i & \mathcal{A}_v) are defined as:

$$\mathcal{A}_i = \frac{\text{Correctly Classified}}{\text{Total Instances}}; \quad \mathcal{A}_v = \frac{(1 - \text{FAR}) + (1 - \text{FRR})}{2}$$

When a lower number of signatures are used for training, a sharp decline appears in recognition performance while a modest decrease in verification performance is noted. For example, when just two signatures are used in the training set, overall I&V rates for {SVC, SigComp} datasets get reduced to {56%, 61%} and {79%, 77%} respectively. This can be

TABLE II. PERFORMANCE OF PROPOSED SCHEME ON TWO PUBLICLY AVAILABLE HANDWRITTEN SIGNATURE DATASETS

Datasets	Sub-Corpus	Identification Rate	Verification Rate		
			FRR	FAR	Accuracy
SVC 2004	Task 1	99.28%	4.9 %	5.3 %	94.87 %
	Task 2	98.16%	5.7 %	5.9 %	94.22 %
	Overall	98.63%			94.56 %
SigComp 2011	Chinese	99.70%	6.2 %	7.4 %	93.21 %
	Dutch	99.72%	3.9 %	5.0 %	95.54 %
	Overall	99.71%			94.38 %

attributed to the fact that sparse coding based recognition requires an adequate number of training samples for good accuracy.

We tested the ability of our scheme to reject signatures that do not belong to the training set. For this purpose, we generated a dictionary containing users from SVC dataset and tested it with signatures belonging to SigComp dataset. The same procedure is followed for evaluating rejection rates on SigComp trained dictionary for signature samples from SVC. In each case, a threshold is learned from the training set using a max-margin approach. We get 100% rejection rates by thresholding the net residual from recovered signal for each test signature.

IV. CONCLUSION

Signature verification based systems require user-name along with the handwritten signatures which make them cumbersome to use in comparison to other biometrics (like face, fingerprint etc.) that tackle both I&V problems. In this work we have proposed a framework that requires only signatures and automatically performs the task of both person I&V without the need of any other information. Sparse representation and compressed sensing principles are used in this work to identify the correct user. SRI can be successfully used to reject signatures that do not belong to the training set. In the second stage, CCA is used to align the signature data with reference template and then a time warping distance measure is used to verify signatures. In this way, both elastic and geometric transformations are tackled while performing verification. We have achieved average I&V rates of 99.17% and 94.47% respectively.

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TABLE III. STATE OF THE ART PERFORMANCES ON SVC AND SIGCOMP DATASETS [15] [16] (BEST RESULTS IN EACH CATEGORY ARE SHOWN IN BOLD-ALL ACCURACIES ARE IN %)

Method	SVC 2004			Method	SigComp 2011		
	Task 1	Task 2	Overall		Chinese	Dutch	Overall
Sabancı	94.50	93.04	93.77	Xyzmo-2	93.17	96.35	94.76
UPM-b	88.01	93.10	90.55	Xyzmo-1	93.17	94.27	93.72
UPM-c	88.02	93.09	90.55	Sabancı	84.81	93.49	89.15
Ours	94.87	94.22	94.56	Ours	93.21	95.54	94.38

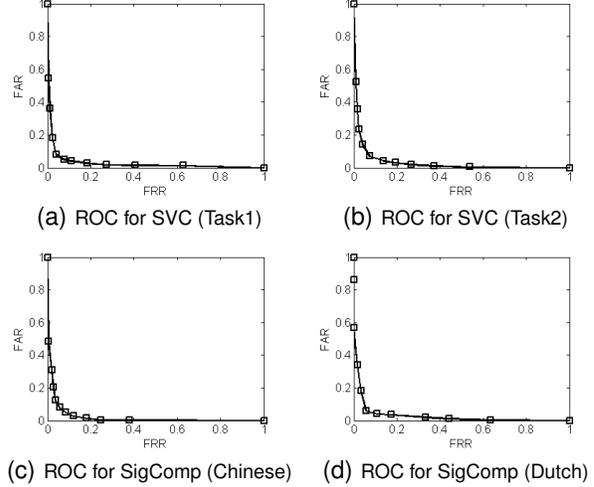


Fig. 2. Receiver Operating Curves are separately plotted for both subcorpus in each of SVC and SigComp Datasets

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